Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Two tangents can be drawn from U.

If we had used the point where the internal common tangent cuts CC' we would have determined two other circles, four in all, satisfying the condition.

Also solved by *PROF. F. E. MILLER*, and *CHAS. C. CROSS*. Prof. Cooper D. Schmitt did not solve the problem but gave several references where solutions are given. Prof. J. Scheffer gave a short historical note on the problem.

In a future issue of the MONTHLY, we expect to publish a somewhat exhaustive discussion of this very interesting problem.

90. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

The bisectors of the angles of the opposite sides (produced) of an inscribed quadrilateral cut the sides at the angular points of a rhombus.

Solution by G. I. HOPKINS, A. M., Professor of Mathematics in High School, Manchester, N. H.; J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.; J. W. SCROGGS, Principal of Rogers Academy, Rogers, Ark.; NELSON L. RORAY, Professor of Mathematics, South Jersey Institute, Bridgeton, N. J.; HENRY N. DAVIS, Providence, R. I.; ALOIS F. KOVARIK, Professor of Mathematics, Decorah Institute, Decorah, Ia.; and the PROPOSER.

In the triangles AEK and LEC, $\angle AEG = \angle LEC$, $\angle EAK = \angle LCE$.

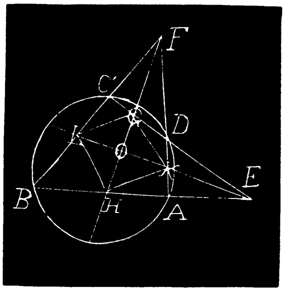
$\therefore$ $\angle EKA = \angle ELC$. $\therefore$ $\angle DKL = \angle CLK$.

$\therefore$ FH is perpendicular to KL at its middle point. Similarly, EL is perpendicular to GH at its middle point.

$\therefore$ In the right triangles KOG, KOH, $KO = KO$, $GO = OH$.

$\therefore$ $KG = KH$. Similarly $KG = GL = LH$.

$\therefore$ $KGLH$ is a rhombus.



This problem was also solved in a similar manner by *E. T. BUSH* and *S. L. ROWAN*, of the Freshman Class of the University of Mississippi; *P. S. BERG*, *W. H. DRANE*, *F. R. HONEY*, *E. R. ROBBINS*, *B. F. SINE*, *J. SCHEFFER*, and *J. F. TRAVIS*.

CALCULUS.

70. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Graduate Fellow in Mathematics in Vanderbilt University, P. O., Lynnville, Tenn.

$$\text{Prove } \int_0^\infty \frac{\cos ax}{1+x^{2n}}dx = -i\frac{\pi}{2n}\sum_{r=1}^{n} \omega^{2r-1}e^{ai\omega^{2r-1}}$$

where n is an integer, a is positive, and ω is $e^{i(\pi\,2n)}$.

Solution by the PROPOSER.

Consider the integral $\int \frac{e^{iay}}{1+y^{2n}}dy$, where a is real and positive. The poles given by $y^{2n} = -1$ or $z = i^{1/n} = \cos(\pi/2n) + i\sin(\pi/2n) = e^{i(\pi/2n)} = \omega$ (say).

It is evident that all the roots of $y^n = i$ are given by ω^{2r-1}, where r may have the values 1, 2, 3, ...n.

Hence $y=\omega^{2r-1}$. About the origin O as center describe a semi-circle ABD with a very large radius, limited by the axis of X (see figure.) About the poles c, which correspond to the points $y^n=i$, describe circles with a very small radius ρ. The proposed function being holomorphic in the portion of the plane lying between the circumferences ρ and the contour $ABDA$, the integrals

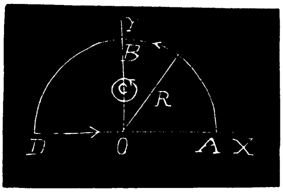
$$\int \frac{e^{iay}}{1+y^{2n}}dy\cdot$$

relative to the circles and the boundary $ABDA$ are equal. For points on the circles ρ $y=\omega^{2r-1}+\rho e^{i\theta}$, and the integral becomes

$$\int_0^{2\pi} \frac{e^{ia(\omega^{2r-1}+\rho e^{i\theta})}}{1+(\omega^{2r-1}+\rho e^{i\theta})} i\rho e^{i\theta}d\theta, =i\int_0^{2\pi} \frac{e^{ai\omega^{2r-1}}}{2n\omega^{(2r-1)(2n-1)}}d\theta$$

(when ρ becomes infinitesimal),

$$=-i\int_0^{2\pi} \frac{\omega^{2r-1}e^{ai\omega^{2r-1}}}{2n}d\theta, =-i\frac{\pi}{n}\sum_{r=1}^{n}\omega^{2r-1}e^{ai\omega^{2r-1}}$$



For points on the semi-circle ABD, $y=R\cos\theta+iR\sin\theta$, and the integral becomes,

$$i\int_0^{\pi} \frac{Re^{i\theta}e^{aR(\cos\theta+i\sin\theta)}}{1+R^{2n}e^{i2n\theta}}d\theta,$$

which is evidently equal to zero when $R=\infty$, and we have left the integral along DA, which is

$$\int_{-\infty}^{\infty} \frac{e^{aix}}{1+x^{2n}}dx=-i\frac{\pi}{n}\sum_{r=1}^{n}\omega^{2r-1}e^{ai\,\omega^{2r-1}}=\int_{-\infty}^{0}\frac{e^{aix}dx}{1+x^{2n}}+\int_0^{\infty}\frac{e^{aix}dx}{1+x^{2n}}$$

$$=\int_0^{\infty}\frac{e^{aix}+e^{-aix}}{1+x^{2n}}dx, =2\int_0^{\infty}\frac{\cos ax}{1+x^{2n}}dx.$$

Therefore $\displaystyle\int_0^{\infty}\frac{\cos ax dx}{1+x^{2n}}=-i\frac{\pi}{2n}\sum_{r=1}^{n}\omega^{2r-1}e^{ai\omega^{2r-1}}$

Is this result correst? Forsyth gives, on page 41 of his *Theory of Functions*, the integral

$$\int_{-\infty}^{\infty}\frac{\cos ax dx}{1+x^{2n}}=-i\frac{\pi}{2n}\sum_{r=1}^{n}\omega^{2r-1}e^{ai\omega^{2r-1}}$$

[Prof. Zerr remarks that the result is correct, as is easily seen from the following:

Let $\cos ax/(1+x^{2n})=f(x)$.

$$\int_{-\infty}^{\infty} f(x)dx=\int_0^{\infty} f(x)dx+\int_{-\infty}^{0} f(x)dx=\int_0^{\infty} f(x)dx-\int_{\infty}^{0} f(x)dx=\int_0^{\infty} f(x)dx$$

$$+\int_0^{\infty} f(x)dx=2\int_0^{\infty} f(x)dx.$$

$$\therefore \int_0^{\infty} f(x)dx=\tfrac{1}{2}\int_{-\infty}^{\infty} f(x)dx,=-i\frac{\pi}{2n}\sum_{r=1}^{n}\omega^{2r-1}e^{ai\omega^{2r-1}}.]\,.$$

71. **Proposed by J. C. CORBIN, Pine Bluff, Ark.**

Form the differential equation of the third order, of which $y=c_1e^{2x}+c_2e^{-3x}+c_3e^x$ is the complete primitive.

I. Solution by EDGAR ODELL LOVETT, Ph. D., Princeton University, Princeton, N. J.

1°. This problem is a familiar one to students of differential equations. The original primitive together with the results of three successive differentiations, may be written

$$\begin{aligned}
&y-e^{2x}c_1-e^{-3x}c_2-e^{x}c_3=0,\\
&y'-2e^{2x}c_1+3e^{-3x}c_2-e^{x}c_3=0,\\
&y''-4e^{2x}c_1-9e^{-3x}c_2-e^{x}c_3=0,\\
&y'''-8e^{2x}c_1+27e^{-3x}c_2-e^{x}c_3=0\,;
\end{aligned}$$

where $y'\equiv\dfrac{dy}{dx}$, $y''\equiv\dfrac{d^2y}{dx^2}$, $y'''\equiv\dfrac{d^3y}{dx^3}$.

The above is a system of linear and homogeneous equations in the quantities 1, $e^{2x}c_1$, $e^{-3x}c_2$, and $e^{x}c_3$, hence the determinant of their coefficients vanishes, that is

$$\begin{vmatrix} y & 1 & 1 & 1\\ y' & 2 & -3 & 1\\ y'' & 4 & 9 & 1\\ y''' & 8 & -27 & 1\end{vmatrix}\equiv\begin{vmatrix} y & 1 & 1 & 1\\ y'-y & 1 & -4 & 0\\ y''-y & 3 & 8 & 0\\ y'''-y & 7 & -28 & 0\end{vmatrix}\equiv4\begin{vmatrix} y'-y & 1 & 1\\ y''-y & 3 & -2\\ y'''-y & 7 & 7\end{vmatrix}\equiv0\,;$$

whence

$$\begin{vmatrix} y'-y & 1 & 0\\ y''-y & 3 & -2\\ y'''-y & 7 & 0\end{vmatrix}\equiv2\begin{vmatrix} y'-y & 1\\ y'''-y & 7\end{vmatrix}=0\,;$$

or finally

$$y'''-7y'+6y=0$$

is the differential equation of the third order whose complete primitive is